Stability Analysis in a Pollution Defensive Model with Two Time Delays

H.X. Yao[1], Rui Liu[1,2], Kai Liu[1]

[1]Faculty of Science, Jiangsu University, Zhenjiang, Jiangsu Province, China, 212013
hxyao@ujs.edu.cn lr_smile@163.cn l22k77@163.com

Abstract: In this paper a three-dimensional pollution defensive model with two time delays in the Chemical Industrial Area (CIA) is considered. The model is based on the interaction among chemical firms, pollution and capital stock in the CIA. The profit from chemical products is used for both defensive expenditure and an increase in capital stock. It shows that Hopf bifurcation occurs at the equilibrium point when the time delay reaches that point or the pollution defensive against the impact of chemical production is insufficient. In other words, if we do not guard against pollution sufficiently or control the production of chemical firms, it will lead to destabilization. Numerical simulations are given to illustrate the results.

Keywords: Stability; Hopf bifurcation; Time delay; Pollution defensive

1. Introduction

Recent literature has demonstrated complicated relations between environment and capital. Becker [1] has examined the trade-off between capital accumulation and environmental quality through an analysis of regular maximum programs in the framework developed by Brock. The result is a constant utility path supported by government-imposed effluent charges and environmental rentals, sufficient conditions for a regular maximum path to satisfy the Hartwick Rule in calculating the combination of capital and environmental quality left for future generations. Similarly, a dynamic general equilibrium model based on environmental resources has been developed in a small open economy [2]. The result shows that the level of consumption and the fraction of income devoted to maximize the long-run welfare depend on both consumption level and environmental quality. Furthermore the possibility that both consumption and production affect the environment has been taken into account [3]. Consider the endogenous-growth model with physical and human capital accumulation [4]: the result shows that parameters on preferences, technologies and depreciation rates as well as fiscal policy are relevant to determine qualitatively the dynamic behavior of the economy. This paper considers a three-dimensional environmental defensive expenditures' model with time-delay bases demonstrating the interaction among visitors, quality of ecosystem goods and capital in protected areas [5].

This paper formulates a simple model with a unique positive equilibrium (when it exists) among the variable of state considered;moreover, this equilibrium is always stable. The aim of this work is to analyze how the stability of the equilibrium changes when two time-delays are considered because the dynamics of the pollution and capital stock at time t depend on the profits of chemical production. In this model, we can see how such stability changes give rise to the Hopf bifurcation when time-delay passes through a sequence of critical values. The Hopf bifurcation allows us to find the existence of a region of instability in the neighborhood of a fixed point where the manager of the CIA can stabilize the system if the time-delay is sufficiently short, but the model will become unstable when time delay is too long.

This paper is organized as follows: in Section 2 the model is presented; in Section 3 the fixed point, stability analysis and the existence of Hopf bifurcation are presented; and in Section 4, numerical simulations are presented.

2. The Model

The model, referring to the generic Chemical Industrial Area (CIA), has three variables: the production of chemical firm $V(t)$ in the CIA, the pollution $P(t)$ and the capital stock $K(t)$ intended as structures.

2.1 The production V :

$$\dot{V}(t) = aK(t) - bP^2(t) - cV^2(t) \tag{1}$$

That parameter $a > 0$ represents the production of unit capital stock, and $P(t)$ the current pollution. With the pollution growing, the production will decease; hence the coefficient is b . The production of the CIA never grows continuously, and then, c represents recession coefficient.

2.2 The pollution P :

Following Becker [1] and Cazzavillan and Musu [6], the pollution is defined as less than the maximum tolerable pollution $\bar{P}$: that is, $0 \le P(t) \le \bar{P}$

We assume that a constant proportion $0 < r < 1$ of the pollution is assimilated at each time t . Moreover, supposing that the pollution P increases in proportion to the production V of the CIA, the increase coefficient is d . When no resources are devoted to abatement expenditure, the CIA influences the pollution only by controlling production

$V(t)$.

$$\dot{P}(t)=dV(t)-rP(t) \quad (2)$$

Chemical production makes a positive impact on this pollution. Therefore, chemical firms use a share $0<\rho<1$ of their profit to defend the environmental resources in the CIA in which the profit of one unit product is n .This expenditure is directly proportional to the pollution. Therefore, the dynamics of the pollution is

$$\dot{P}(t)=dV(t)-rP(t)-mn\rho V(t) \quad (3)$$

The parameter $m>0$ is a constant parameter determining how an additional unit of defensive expenditure decreases the pollution.

2.3 The capital stock K :

The other share $(1-\rho)$ of the total profit of chemical product is used to increase capital stock

$$\dot{K}(t)=n(1-\rho)V(t)-\delta K(t) \quad (4)$$

Capital stock is assumed to depreciate at the rate $\delta>0$. Considering two time delays, the model is formulated as follows:

$$\begin{cases}\dot{V}(t)=aK(t)-bP^2(t-\tau_2)-cV^2(t)\\ \dot{P}(t)=dV(t-\tau_1)-rP(t-\tau_2)-mn\rho V(t-\tau_1)\\ \dot{K}(t)=n(1-\rho)V(t-\tau_1)-\delta K(t)\end{cases} \quad (5)$$

3. Qualitative Behavior of the Model

3.1 The equilibrium point:

It's easy to know that the equilibrium points are these:

$$F^0(0,0,0), \quad F^*(V^*,P^*,K^*)$$

where

$$V^*=\frac{anr^2(1-\rho)}{b\delta(d-mn\rho)^2+c\delta r}, P^*=\frac{d-mn\rho}{r}V^*, K^*=\frac{n(1-\rho)}{\delta}V^*$$

Obviously, V^*, E^* and K^* are only determined by the model (5), and they are always positive, if $(d-mn\rho)>0$.

3.2 Stability analysis:

The linearization of the model in the neighborhood of the positive equilibrium F^* yields:

$$\begin{pmatrix}\dot{V}(t)\\ \dot{P}(t)\\ \dot{K}(t)\end{pmatrix}=\eta_1\begin{pmatrix}V(t)-V^*\\ P(t)-P^*\\ K(t)-K^*\end{pmatrix}+\eta_2\begin{pmatrix}V(t-\tau_1)-V^*\\ P(t-\tau_1)-P^*\\ K(t-\tau_1)-K^*\end{pmatrix}+\eta_3\begin{pmatrix}V(t-\tau_2)-V^*\\ P(t-\tau_2)-P^*\\ K(t-\tau_2)-K^*\end{pmatrix} \quad (6)$$

where

$$\eta_1=\begin{pmatrix}0&0&a\\0&0&0\\0&0&-\delta\end{pmatrix}, \eta_2=\begin{pmatrix}-2cV^*&0&0\\(d-mn\rho)e^{-\lambda\tau_1}&0&0\\n(1-\rho)e^{-\lambda\tau_1}&0&0\end{pmatrix}$$

$$\eta_3=\begin{pmatrix}0&-2bP^*e^{-\lambda\tau_2}&0\\0&-re^{-\lambda\tau_2}&0\\0&0&0\end{pmatrix}$$

So the characteristic equation of model (5) is $det(\lambda I-\eta_1-\eta_2e^{-\lambda\tau_1}-\eta_3e^{-\lambda\tau_2})=0$,

which leads to

$$D(\lambda,\tau_1,\tau_2)=S(\lambda)e^{-\lambda(\tau_1+\tau_2)}+R(\lambda)e^{-\lambda\tau_1}+Q(\lambda)e^{-\lambda\tau_2}+P(\lambda)=0 \quad (7)$$

where

$$S(\lambda)=2bP^*(\lambda+\delta)(d-mn\rho)-anr(1-\rho);$$
$$R(\lambda)=-an\lambda(1-\rho);$$
$$Q(\lambda)=r(\lambda+\delta)(\lambda+2cV^*);$$
$$P(\lambda)=\lambda(\lambda+\delta)(\lambda+2cV^*);$$

3.3 The case $\tau_1=\tau_2=0$:

So, the characteristic polynomial is

$$D(\lambda,0,0)=S(\lambda)+R(\lambda)+Q(\lambda)+P(\lambda)=\lambda^3+A_0\lambda^2+A_1\lambda+A_2=0 \quad (8)$$

where

$$A_0=2cV^*+\delta+r;$$
$$A_1=2cV^*(r+\delta)+2bP^*(d-mn\rho)+r\delta-an(1-\rho);$$
$$A_2=2b\delta P^*(d-mn\rho)-anr(1-\rho)+2cr\delta V^*;$$

According to the Routh-Hurwitz criterion [8], the equilibrium point is stable if, and only if

$$H_1: A_0>0 \quad A_0A_1-A_2>0;$$

3.4 The case $\tau_1\neq0, \tau_2=0$:

Let $\tau_2=0$ in Eq. (7), the characteristic polynomial becomes

$$\begin{aligned}D(\lambda,\tau_1,0)&=[S(\lambda)+R(\lambda)]e^{-\lambda\tau_1}+Q(\lambda)+P(\lambda)\\&=\lambda^3+B_0\lambda^2+B_1\lambda+B_2+[B_3\lambda+B_4]e^{-\lambda\tau_1}=0\end{aligned} \quad (9)$$

where

$$B_0=r+\delta+2cV^*;$$
$$B_1=2cV^*(r+\delta)+r\delta;$$
$$B_2=2cr\delta V^*;$$
$$B_3=2bP^*(d-mn\rho)-an(1-\rho);$$
$$B_4=2b\delta P^*(d-mn\rho)-anr(1-\rho);$$

Theorem 1. Eq. (9) has a unique pair of purely imaginary roots if $(B_2+B_4)(B_2-B_4)<0$

Proof: If $\lambda=i\omega, \omega>0$ is a root of (9), separating real and imaginary parts, there will be the following equations:

$$\begin{cases}B_0\omega^2-B_2=B_4\cos\omega\tau+B_3\omega\sin\omega\tau\\ \omega^3-B_1\omega=B_3\omega\cos\omega\tau-B_4\sin\omega\tau\end{cases} \quad (10)$$

Squaring and adding both equations above

$$\omega^6+Q_1\omega^4+Q_2\omega^2+Q_3=0 \quad (11)$$

where

$$Q_1 = B_0^{2} - 2B_1,\ Q_2 = B_1^{2} - 2B\,B_0 - B_2^{2};\ Q_3 = B_3^{2} - B_2$$

leads to:

$$\begin{aligned} Q_1 &= B_0^{2} - 2B_1 \\ &= (\delta + r + 2cV^*)^2 - 2(\delta r + 2crV^* + 2c\delta V^* - an(1-\rho)) \\ &= 4c^2V^{*2} + \delta^2 + r^2 + 2an(1-\rho) > 0 \end{aligned} \tag{12}$$

$$Q_3 = B_2^{2} - B_4^{2} = (B_2 + B_4)(B_2 - B_4) < 0 \tag{13}$$

Then the conditions of this theorem imply that there is a unique positive root ω_0 satisfying Eq. (9). That is, it has a unique pair of purely imaginary roots $\pm i\omega_0$.

From Eq. (10) τ_{1n} can be obtained

$$\tau_{1n} = \frac{1}{\omega_0} cos^{-1} \frac{B_3\omega_0^{4} + (B_0B_4 - B_1B_3)\omega_0^{2} - B_2B_4}{B_4^{2} + B_3^{2}\omega_0^{2}} + \frac{2k\pi}{\omega_0}$$
$$k = 0,1,2,... \tag{14}$$

Theorem 2. If the following conditions

$$H_2 : (B_2 + B_4)(B_2 - B_4) < 0\,;\ B_0(B_1 + B_3) > B_2 + B_4\,;$$
$$(B_1^{2} - 2B_0B_2)B_4^{2} > B_2^{2}B_3^{2}\,;$$

hold, model (5) undergoes Hopf bifurcation at $F^*(V^*, E^*, K^*)$ when $\tau = \tau_{10}$; furthermore, F^* is locally asymptotically stable if $\tau_1 \in [0, \tau_{10})$, but unstable if $\tau_1 > \tau_{10}$.

Proof. It has been proved that, when $\tau_1 = 0$, all roots of Eq. (7) have negative real parts: that is to say, the equilibrium F^* is locally stable for $\tau_1 = 0$. Subsequently, when $\tau_1 < \tau_{10}$, F^* is still stable.

Then, if $Re\left(\frac{d\lambda}{d\tau_1}\right)\Big|_{\tau=\tau_{10}} > 0$, it indicates that when $\tau_1 > \tau_{10}$, at least a characteristic root with a positive real part will exist. According to the conditions of Hopf bifurcation theorem, the periodic solutions will occur when $\tau_1 > \tau_{10}$.

Differentiating Eq. (9) with τ_1, it is as follows:

$$\begin{aligned} &[3\lambda^2 + 2B_0\lambda + B_1 + B_3e^{-\lambda\tau_1} - \tau_1(B_3\lambda + B_4)e^{-\lambda\tau_1}]\frac{d\lambda}{d\tau_1} \\ &= \lambda(B_3\lambda + B_4)e^{-\lambda\tau_1} \end{aligned} \tag{15}$$

that is,

$$\begin{aligned} \left(\frac{d\lambda}{d\tau_1}\right)^{-1} &= \frac{3\lambda^2 + 2B_0\lambda + B_1}{\lambda(B_3\lambda + B_4)e^{-\lambda\tau_1}} - \frac{\tau_1}{\lambda} \\ &= -\frac{3\lambda^2 + 2B_0\lambda + B_1}{\lambda(\lambda^3 + B_0\lambda^2 + B_1\lambda + B_2)} + \frac{B_3}{\lambda(B_3\lambda + B_4)} - \frac{\tau_1}{\lambda} \end{aligned} \tag{16}$$

Thus,

$$Re\left(\frac{d\lambda}{d\tau_1}\right)^{-1}\Bigg|_{\lambda=i\omega} = Re\left(-\frac{B_1 - 3\omega^2 + i2B_0\omega}{i\omega[(B_2 - B_0\omega^2) + i\omega(B_1 - \omega^2)]} + \frac{B_3}{i\omega(iB_3\omega + B_4)}\right)$$
$$= Re\left(\frac{2B_3^{2}\omega^6 + (3B_4^{2} + B_0^{2}B_3^{2} - 2B_1B_3^{2})\omega^4 + (2B_0^{2} - 4B_1)B_4^{2}\omega^2 + (B_1^{2}B_4^{2} - B_2^{2}B_3^{2} - 2B_0B_2B_4^{2})}{[\omega^2(B_1 - \omega^2)^2 + (B_2 - B_0\omega^2)^2][(B_3\omega)^2 + B_4^{2}]}\right)$$

We can rewrite the numerator as follows.

$$\begin{aligned} f(\omega) &= 2B_3^{2}\omega^6 + (3B_4^{2} + B_0^{2}B_3^{2} - 2B_1B_3^{2})\omega^4 \\ &+ (2B_0^{2} - 4B_1)B_4^{2}\omega^2 + (B_1^{2}B_4^{2} - B_2^{2}B_3^{2} - 2B_0B_2B_4^{2}) \end{aligned} \tag{17}$$

Let $\eta = \omega^2$, then

$$\begin{aligned} G(\eta) = f(\omega) &= 2B_3^{2}\eta^3 + (3B_4^{2} + B_0^{2}B_3^{2} - 2B_1B_3^{2})\eta^2 \\ &+ (2B_0^{2} - 4B_1)B_4^{2}\eta + (B_1^{2}B_4^{2} - B_2^{2}B_3^{2} - 2B_0B_2B_4^{2}) \end{aligned} \tag{18}$$

and

$$G'(\eta) = 2[3B_3^{2}\eta^2 + (3B_4^{2} + (B_0^{2} - 2B_1)B_3^{2})\eta + (B_0^{2} - 2B_1)B_4^{2}] \tag{19}$$

for $G'(\eta)$,

$$\begin{aligned} \Delta &= (3B_4^{2} + (B_0^{2} - 2B_1)B_3^{2})^2 - 12(B_0^{2} - 2B_1)B_3^{2}B_4^{2} \\ &= [3B_4^{2} - (B_0^{2} - 2B_1)B_3^{2}]^2 \geq 0 \end{aligned} \tag{20}$$

G' has two real roots, which take the for

$$\eta_1 = \frac{-(3B_4^{2} + (B_0^{2} - 2B_1)B_3^{2}) + \sqrt{\Delta}}{6B_3^{2}} < 0\,,$$

$$\eta_1 = \frac{-(3B_4^{2} + (B_0^{2} - 2B_1)B_3^{2}) - \sqrt{\Delta}}{6B_3^{2}} < 0$$

From the above, clearly $G'(\eta)$ increases monotonously in $(\eta_1, +\infty)$. As far as concerned that $f(0) = B_1^{2}B_4^{2} - B_2^{2}B_3^{2} - 2B_0B_2B_4^{2} > 0$, there will be $f(\omega) > 0$, for $\omega > 0$, and it will have:

$$sign\frac{d(Re\,\lambda(\tau_1))}{d\tau_1}\Bigg|_{\tau_1=\tau_{1n}} = sign\,Re\left(\frac{d\lambda}{d\tau_1}\right)^{-1}\Bigg|_{\lambda=i\omega} > 0 \tag{21}$$

Theorem2 states that when these conditions obtain, the Hopf bifurcation will occur while τ_{10} is the minimum τ_{1n} at which the real parts of these roots are zero. That is to say, the model undergoes Hopf bifurcation at the equilibrium F^* when $\tau_1 = \tau_{10}$, and, regarding the impact of production on pollution, the defensive expenditures is not elevated. In other words, if the pollution is not sufficiently decreased, it will reach one destabilization at the fixed points when $\tau_1 > \tau_{10}$.

Theorem 3: For Eq. (9), it can refer to:
If H_1 and H_2 hold, when $\tau_1 \in [0, \tau_{10})$ all roots of Eq. (9) have negative real parts, and when $\tau_1 > \tau_{10}$ Eq. (9) will have at least one root with positive real part.

Proof: As H_1 and H_2 hold, then the equilibrium of the Eq.(9) is stable and Eq.(9) has complex roots with negative real parts for $\tau_1 = 0$, and also for $\tau_1 = \tau_{10}$. Eq.(9) has purely imaginary roots, and the real parts of the root changes continuously with the increase of τ_1 because of $sign\frac{d(Re\,\lambda(\tau_1))}{d\tau_1}\Big|_{\tau_1=\tau_{1n}} > 0$, so for $\tau_1 \in [0, \tau_{10})$ all roots of Eq.(9) have negative real parts and Eq.(9) has at least one root with positive real parts when $\tau_1 > \tau_{10}$.

3.5 The case $\tau_1 \neq 0, \tau_2 \neq 0$:

Next, we return to the Eq. (7) with $\tau_2 > 0$ and τ_1 in stable regions. Regard τ_2 as a parameter following Ruan and Wei [10]

Theorem 4: If all roots of Eq. (9) have negative parts for $\tau_1 > 0$, then there will exist a $\tau_2^*(\tau_1) > 0$, subject to all roots of Eq. (7), and it will have negative real parts when $0 \le \tau_2 < \tau_2^*(\tau_1)$.

Proof. Following the theorem2.1 of Ruan and Wei, the left of Eq.(7)is analytic in λ and τ_2, and when τ_2 varies, the sum of the multiplicities of zeros of the left of Eq.(7) in the open half-plane will change only if a zero is on, or crosses, the imaginary axis.

Theorem 5: Assume H_1 holds true; if H_2 holds, there exists $0 < \tau^*_{10} < \tau_{10}$ and $\tau_2 = \tau_2^*(\tau_1)$, then for any $\tau_1 \in [0, \tau^*_{10})$,the equilibrium of model (5) is locally asymptotically stable when $\tau_2 \in [0, \tau_2^*(\tau_1))$.

Proof. According to Theorem3 and Theorem4, there will be a result.

It's clear that the Hopf bifurcation occurs at $\tau_2^*(\tau_1)$ if it holds the conditions of Theorem 4 or Theorem 5 and, also, there may be a lot of stability-switches. If τ_1 is in an unstable region, there may not exist $\tau_2^*(\tau_1)$ which makes the model (5) stable if $0 \le \tau_2 < \tau_2^*(\tau_1)$, but unstable if $\tau_2 > \tau_1^*(\tau_1)$.

4. Numerical Simulation

This section shows some numerical simulations at different value of τ_1 and τ_2.

Considering system (5) with the following parameters $a = 4$, $b = 0.5$, $c = 0.1$, $d = 0.9$, $r = 0.1$, $n = 1$, $\rho = 0.8$, $m = 1$, $\delta = 0.1$, initial values $V = 2, P = 1, K = 3$, the conditions of Theorem 1,2 hold. Supposing $\rho = 0.8$, the fixed point is $F^* = (13.75, 13.75, 27.50)$.

4.1 The case $\tau_1 \ne 0, \tau_2 = 0$

There is $\omega_0 = 0.1445$, $\tau_{10} = 12.6813$ period $T = 43.4782$.

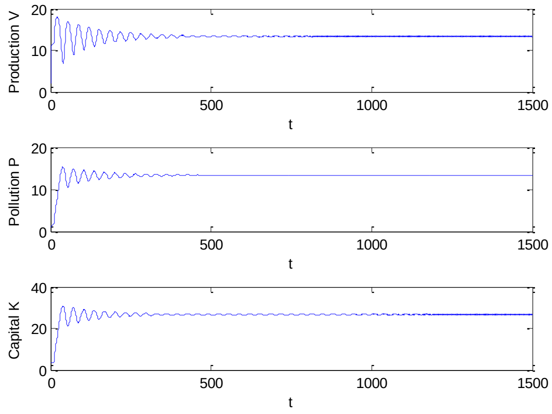

Fig 1when $\tau_2 = 0$ and $\tau_1 = 10 < \tau_{10}$

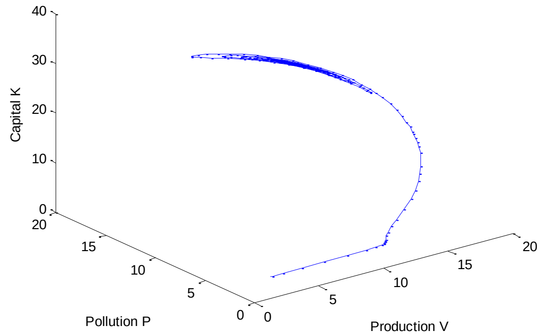

Fig2:when $\tau_2 = 0$ and $\tau_1 = 10 < \tau_{10}$

Figures1,2 show that when $\tau_2 = 0$ and $\tau_1 = 10 < \tau_{10}$, the chemical production, the pollution and the capital stock tend to be stable.

Figures3,4 show that when $\tau_2 = 0$ and $\tau_1 = 14 > \tau_{10}$, the chemical production, the pollution and the capital stock tend to be periodic solutions.

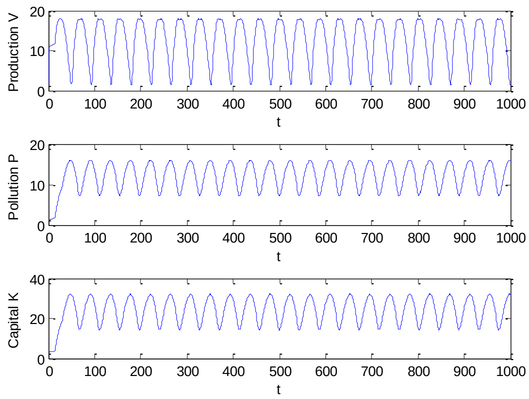

Fig3: when $\tau_2 = 0$ and $\tau_1 = 14 > \tau_{10}$

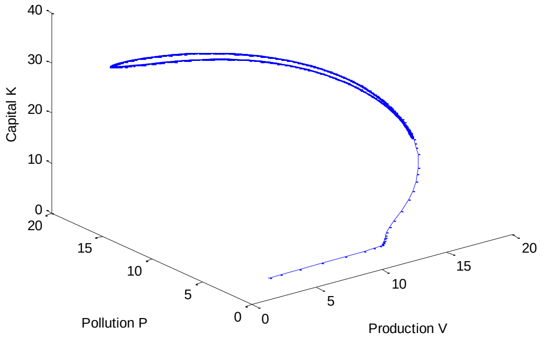

Fig4: when $\tau_2 = 0$ and $\tau_1 = 14 > \tau_{10}$

4.2 The case $\tau_1 \neq 0, \tau_2 \neq 0$

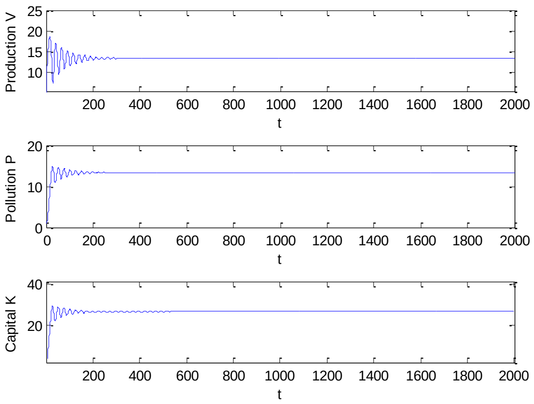

Fig5: when $\tau_1 = 5.7 < \tau^*_{10}$ and $\tau_2 = 0.7 < \tau_2^*(\tau_1)$

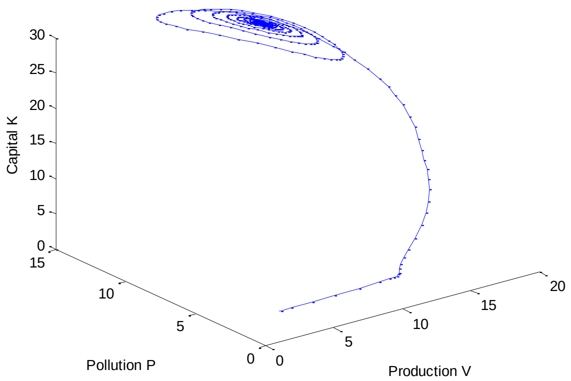

Fig6:when $\tau_1 = 5.7 < \tau^*_{10}$ and $\tau_2 = 0.7 < \tau_2^*(\tau_1)$

Figures5,6 show that when $\tau_1 = 5.7 < \tau^*_{10}$ and $\tau_2 = 0.7 < \tau_2^*(\tau_1)$, the chemical production, the pollution and the capital stock tend to be stable.

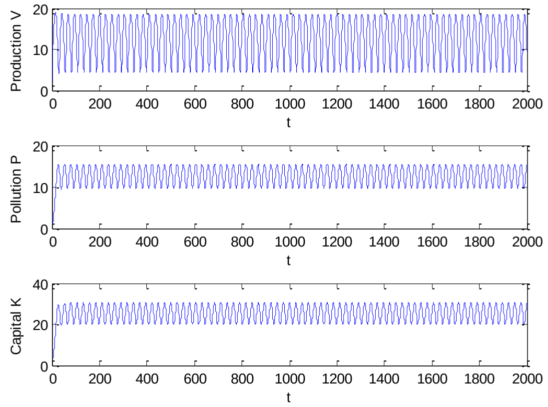

Fig7: when $\tau_1 = 5.7 < \tau^*_{10}$ and $\tau_2 = 1.0081 > \tau_2^*(\tau_1)$

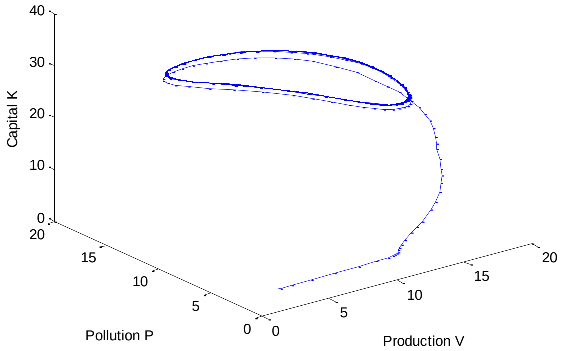

Fig8: when $\tau_1 = 5.7 < \tau^*_{10}$ and $\tau_2 = 1.0081 > \tau_2^*(\tau_1)$

5. Conclusion

The present work, starting from a simple model with a positive equilibrium, shows that a delay may generate instability and, as a consequence, problems in the sustainability of the CIA's decision if the condition $(d - mn\rho) < 0$ occurs: that is, the pollution defensive expenditure is not elevated sufficiently. Furthermore, if the conditions (1) $(C+E)(C-E) < 0$; (2) $A(B+D) > C+E$; (3) $(B^2 - 2AC)E^2 > C^2D^2$ hold, $\tau_1 > \tau_{10}$, the Hopf bifurcation occurs but, then, if $\tau_2 > 0$ and τ_1 exist in stable regions, it's clear that Hopf bifurcation will occur at $\tau_2^*(\tau_1)$ if the conditions of Theorem 4 or Theorem 5 hold. Further developments can be identified or analyzed in a model with two variable delays.

References

[1] Rober A. Becker. Intergenerational equity: The capital-environment trade-off [J]. J Environ Econ Manage. 1982; 9:165-85.

[2] Rinaldo Brau, Alessandro Lanza, Stefano Usai. Tourism and sustainable economic development: macroeconomic

model and empirical methods [M]. New York:William Partt House, 2008.
[3] Carlo Carraro, Domenico Siniscalco. New direction in the economic theory of the environment [M], London: Cambridge University Press, 1997.
[4] Salvador Ortigueira. Fiscal policy in an endogenous growth model with human capital accumulation [J]. Journal of Monetary Economics, 1998, 45(2): 323-335
[5] Paolo Russu. Hopf bifurcation in an environmental defensive expenditures model with time
(6): 991 - 1003.
[8] Hale J K. Theory of Functional Differential Equations
[M]. New York: Spring2 Verlag, 1977.
[9] E.Beretta,Y.Kuang, Geomitric stability switch criteria in delay differential models with delay dependent parameters [J]. SIAM J.Math.Anal.33(2002) 1144-1165.
[10] S.Ruan,J.Wei, On the Zeros of transcendental functions with applications to stability of delay differential equations with two delays [J]. Dyn.Contin. Discrete Impuls. Syst. Ser.A Math. Anal. 10(2003) 863-874.
Delay [J]. Chao, Solitons & Fractals. 2009; 42:3147-59.
[6] Cazzavillan G, Musu I. Transitional dynamics and uniqueness of the balanced-growth path in
a simple model of endogenous growth with an environmental asset [J]. FEEM Working Paper 62.2001; 2001.
[7] Freedman H I, Sree Hari Rao V. The Trade 2 off Between Mutual Interference and Time Lags
in Predator 2 prey Models [J]. Bull Math Biol, 1983, 45

Author Biographies

H.X.Yao male,Yangzhou Jiangsu Province of China, professor,,major research direction is the complexity of economic systems modeling and anslysis

Liu Rui famle,Shangqiu,Henan Province of China ,Master,research direction: the complexity of the financial system and Chaos Control.

Liu Kai male, Yantai,Shandong Province of China,Master,research direction: the complexity of the financial system and Chaos Contro